# Inverse analysis for estimating the timewise and spacewise variation of the wall heat flux in a parallel plate channel

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## Nomenclature

$d$ = direction of descent given by equation (6b)
$e_{RMS}$ = RMS error as defined by equation (21)
$J$ = functional defined by equation (5)
$k$ = thermal conductivity
$N$ = total number of measurements
$q$ = heat flux
$S$ = number of sensors
$t$ = time
$T$ = temperature
$u$ = velocity
$x$ = axial co-ordinate
$y$ = transversal co-ordinate
$Y$ = measured temperature
$w$ = channel half-width

*Greek symbols*
$\alpha$ = thermal diffusivity
$\beta$ = search step size given by equation (9)
$\Delta T$ = sensitivity function satisfying problem (7)
$\gamma$ = conjugation coefficient given by equation (6c)
$\varepsilon$ = tolerance
$\lambda$ = Lagrange multiplier satisfying problem (12)
$\sigma$ = standard deviation of the measurements

*Superscripts*
$*$ = dimensional variables
$k$ = number of iterations

*Subscripts*
0 = reference or initial value
ex = exact quantity
est = estimated quantity
$f$ = final value
$i$ = sensor number
$m$ = mean value

## Introduction

Inverse heat conduction problems concerned with the estimation of physical properties, boundary or initial conditions, or geometric characteristics of a heated body have been generally treated in the literature. Excellent books are available for such classes of inverse heat transfer problems, including, among others, those by Beck *et al.* (1985), Murio (1993) and Ozisik (1993).

More recently, inverse convection heat transfer problems have gained the attention of different groups. Moutsoglou (1989) has used Beck's sequential function estimation algorithm (Beck *et al.*, 1985) to estimate the steady-state heat flux distribution at the wall of a vertical parallel plate channel, in a mixed convection problem. This author (Moutsoglou, 1990) has also applied a whole domain regularization technique (Beck *et al.*, 1985) to solve basically the same inverse problem, but for forced convection. The steady-state inlet temperature

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distribution in laminar forced convection has been estimated by Raghunath (1993). The conjugate gradient method with adjoint equation has been applied by Huang and Ozisik (1992) to the estimation of the steady-state wall heat flux in hydrodynamically developed laminar flow through a parallel plate duct. The timewise varying inlet temperature in similar flow conditions has been estimated by Bokar and Ozisik (1995) by also applying the conjugate gradient method with adjoint equation.

In this paper, we use the conjugate gradient method with adjoint equation to estimate the timewise and spacewise variation of the wall heat flux in a parallel plate channel, under laminar and hydrodynamically developed flow conditions. This is a powerful iterative method, which can be applied to linear (Alifanov, 1974; Bokar and Ozisik, 1995; Huang and Ozisik, 1992; Jarny *et al.*, 1991), as well as to non-linear inverse problems (Orlande and Ozisik, 1994).

We use here a function estimation approach, i.e. no information regarding the functional form of the unknown heat flux is considered available for the inverse analysis. The accuracy of the present solution approach is assessed by using simulated transient temperature measurements of several sensors located at appropriate locations inside the channel. The most difficult functions to be recovered by an inverse analysis are those containing sharp corners and discontinuities. The present approach is verified to be sufficiently accurate for such strict conditions.

## Direct problem

The physical problem considered here is the laminar hydrodynamically developed flow between parallel plates of a fluid with constant properties. The inlet temperature is maintained at a constant value $T_0^*$, which is also assumed to be the initial fluid temperature. For times greater than zero, the plates are subjected to a time and space-dependent heat flux, as illustrated in Figure 1.

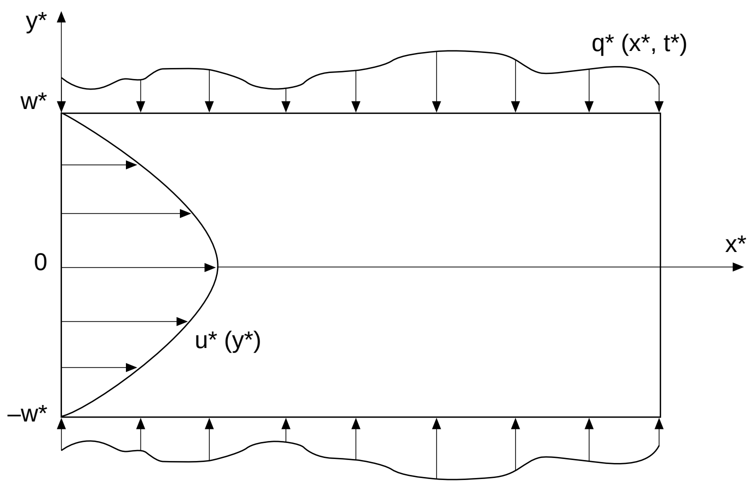


**Figure 1.**
Physical problem

By taking into account the symmetry with respect to the x-axis and neglecting conduction along the flow direction, the mathematical formulation of this problem in dimensionless form is given by:

$$\frac{\partial T}{\partial t} + u(y)\frac{\partial T}{\partial x} = \frac{\partial^2 T}{\partial y^2} \qquad \text{in } 0 < y < 1, x > 0, \text{ for } t > 0 \tag{1a}$$

$$\frac{\partial T}{\partial y} = 0 \qquad \text{at } y = 0, x > 0, \text{ for } t > 0 \tag{1b}$$

$$\frac{\partial T}{\partial y} = q(x,t) \qquad \text{at } y = 1, x > 0, \text{ for } t > 0 \tag{1c}$$

$$T = 0 \qquad \text{at } x = 0, 0 < y < 1, \text{ for } t > 0 \tag{1d}$$

$$T = 0 \qquad \text{for } t = 0, \text{ in } 0 < y < 1, x > 0 \tag{1e}$$

where the following dimensionless groups are introduced:

$$y = \frac{y^*}{w^*};\quad x = \frac{\alpha^* x^*}{u_m^* w^{*2}};\quad T = \frac{T^* - T_0^*}{\frac{q_0^* w^*}{k^*}};\quad t = \frac{\alpha^*}{w^{*2}} t^*; \tag{2a-d}$$

$$u(y) = \frac{u^*(y^*)}{u_m^*} = \frac{3}{2}\left[1 - \left(\frac{y^*}{w^*}\right)^2\right] \tag{2e}$$

$\alpha^*$ and $k^*$ are the fluid thermal diffusivity and conductivity, respectively, $w^*$ is the channel half-width and $u_m^*$ is the mean fluid velocity. The wall heat flux is written as

$$q^*(x^*, t^*) = q_0^* q(x,t) \tag{3}$$

where $q_0^*$ is a constant reference value with units of heat flux and q(x, t) is a dimensionless function of x and t. The superscript "*" above denotes dimensional variables.

The direct problem given by equations (1) is concerned with the determination of the temperature field of the fluid inside the channel, when the boundary heat flux q(x, t) at y = 1 is known.

**Inverse problem**

For the inverse problem, the heat flux q(x, t) at y = 1 is considered to be unknown and is to be estimated by using the transient readings of S temperature sensors located inside the channel. We assume that no information is available regarding the functional form of the unknown wall heat flux, except that it belongs to the space of square integrable functions in $(0, t_f)$ $(0, x_f)$, i.e.

$$\int_0^{t_f} \int_0^{x_f} [q(x,t)]^2 \, dx dt < \infty \tag{4}$$

where $t_f$ is the duration of the experiment and $x_f$ is the length of the test-section in the channel.

The solution of such inverse problem is obtained by minimizing the following functional,

$$J[q(x,t)] = \int_{t=0}^{t_f} \sum_{i=1}^{S} \{T[x_i, y_i, t; q(x,t)] - Y_i(t)\}^2 dt \tag{5}$$

where $Y_i$ is the measured temperature at the sensor location $(x_i, y_i)$ inside the channel and $T[x_i, y_i, t; q(x, t)]$ is the estimated temperature at the same location. Such estimated temperature is obtained from the solution of the direct problem given by equations (1), by using an estimate for the unknown heat flux $q(x, t)$.

The minimization of the functional given by equation (5) is obtained by utilizing the conjugate gradient method, as described next.

## Conjugate gradient method of minimization

The iterative algorithm of the conjugate gradient method, as applied to the estimation of the unknown heat flux $q(x, t)$ is given by Jarny *et al.*, 1991:

$$q^{k+1}(x,t) = q^k(x,t) - \beta^k d^k(x,t) \tag{6a}$$

where the superscript k denotes the number of iterations.

The direction of descent $d^k(x, t)$ is obtained as a conjugation of the gradient direction and of the previous direction of descent as:

$$d^k(x,t) = J'[q^k(x,t)] + \gamma^k d^{k-1}(x,t) \tag{6b}$$

where the conjugation coefficient is obtained from the Fletcher-Reeves expression

$$\gamma^k = \frac{\int_{x=0}^{x_f} \int_{t=0}^{t_f} \{J'[q^k(x,t)]\}^2 dt\,dx}{\int_{x=0}^{x_f} \int_{t=0}^{t_f} \{J'[q^{k-1}(x,t)]\}^2 dt\,dx} \quad \text{for } k = 1,2,3,\ldots \text{ with } \gamma^0 = 0 \tag{6c}$$

In order to implement the iterative algorithm given by equations (6), we need to develop expressions for the search step size $\beta^k$ and for the gradient direction $J'[q^k(x, t)]$, by making use of two auxiliary problems, known as the sensitivity problem and the adjoint problem respectively.

## Sensitivity problem and search step size

The sensitivity problem is obtained by assuming that the heat flux q(x, t) is perturbed by an amount Δq(x, t). Such perturbation in the heat flux causes a perturbation ΔT(x, y, t) in the temperature T(x,y,t). By replacing T(x, y, t) by T(x, y, t) + ΔT(x, y, t) and q(x, t) by q(x, t) + Δq(x, t) in the direct problem given by equations (1), and then subtracting from the resulting expressions the original direct problem, we obtain the following sensitivity problem for the determination of the sensitivity function ΔT(x, y, t):

$$\frac{\partial \Delta T}{\partial t} + u(y)\frac{\partial \Delta T}{\partial x} = \frac{\partial^2 \Delta T}{\partial y^2} \qquad \text{in } 0 < y < 1, x > 0, \text{ for } t > 0 \tag{7a}$$

$$\frac{\partial \Delta T}{\partial y} = 0 \qquad \text{at } y = 0, x > 0, \text{ for } t > 0 \tag{7b}$$

$$\frac{\partial \Delta T}{\partial y} = \Delta q(x,t) \qquad \text{at } y = 1, x > 0, \text{ for } t > 0 \tag{7c}$$

$$\Delta T = 0 \qquad \text{at } x = 0, 0 < y < 1, \text{ for } t > 0 \tag{7d}$$

$$\Delta T = 0 \qquad \text{for } t = 0, \text{ in } 0 < y < 1, x > 0 \tag{7e}$$

An expression for the search step size $\beta^k$ is obtained by minimizing the functional given by equation (5) with respect to $\beta^k$, that is,

$$\min_{\beta^k} J[q^{k+1}(x,t)] = \min_{\beta^k} \int_{t=0}^{t_f} \sum_{i=0}^{S} [T(x_i, y_i, t; q^k - \beta^k d^k) - Y_i(t)]^2 dt \tag{8}$$

By linearizing the estimated temperature $T(x_i, y_i, t; q^k - \beta^k d^k)$ and performing the minimization above, we obtain the search step size as

$$\beta^k = \frac{\int_{t=0}^{t_f} \sum_{i=1}^{S} (T_i - Y_i)\, \Delta T_i(d^k)\, dt}{\int_{t=0}^{t_f} \sum_{i=1}^{S} [\Delta T_i(d^k)]^2\, dt} \tag{9}$$

where $\Delta T_i(d^k)$ is the solution of the sensitivity problem at the sensor position $(x_i, y_i)$, obtained from equations (7) by setting $\Delta q\,(x, t) = d^k(x, t)$.

## Adjoint problem and the gradient equation

In order to obtain the adjoint problem, we multiply the differential equation (1a) of the direct problem by the Lagrange multiplier λ(x, y, t) and integrate over the time and space domains. The resulting expression is then added to equation (5) to obtain the following extended functional:

$$J[q(x,t)] = \int_{t=0}^{t_f}\int_{x=0}^{x_f}\int_{y=0}^{1}\left\{\sum_{i=1}^{S}[T(x,y,t)-Y_i(t)]^2\delta(x-x_i)\delta(y-y_i)\right.$$

*

$$\left.+\left[\frac{\partial T}{\partial t}+u(y)\frac{\partial T}{\partial x}-\frac{\partial^2 T}{\partial y^2}\right]\lambda(x,y,t)\right\}dydxdt \tag{10}$$

where $\delta(\bullet)$ is the Dirac delta function.

We assume that the extended functional given by equation (10) is perturbed by an amount $\Delta J[q(x, t)]$, when the heat flux $q(x, t)$ is perturbed by $\Delta q(x, t)$. An expression for the variation $\Delta J[q(x, t)]$ is obtained by replacing $J[q(x, t)]$ by $J[q(x, t)] + \Delta J[q(x, t)]$ and T(x, *y, t) by $T(x, y, t) + \Delta T(x, y, t)$ in equation (10), and by subtracting the original equation (10) from the resulting expression. We obtain,

$$\Delta J[q(x,t)] = \int_{t=0}^{t_f}\int_{x=0}^{x_f}\int_{y=0}^{1}\left\{\sum_{i=1}^{S}[2(T-Y)\Delta T]\ \delta(x-x_i)\delta(y-y_i)\right.$$

$$\left.+\left[\frac{\partial \Delta T}{\partial t}+u(y)\frac{\partial \Delta T}{\partial x}-\frac{\partial^2 \Delta T}{\partial y^2}\right]\lambda(x,y,t)\right\}dy\,dx\,dt \tag{11}$$

The three terms involving derivatives inside brackets above are integrated by parts with respect to t, x and y respectively. The boundary and initial conditions of the sensitivity problem, equations (7b-7e), are substituted into the resulting expression, which is then allowed to go to zero. The vanishing of the integral terms containing $\Delta T(x, y, t)$ results in the following adjoint problem for the determination of the Lagrange multiplier $\lambda(x, y, t)$:

$$-\frac{\partial \lambda}{\partial t}-u(y)\frac{\partial \lambda}{\partial x}-\frac{\partial^2 \lambda}{\partial y^2}+2\sum_{i=1}^{S}(T-Y)\delta(x-x_i)\delta(y-y_i)=0$$

$$\text{in } 0<y<1, x>0, \text{ for } t>0 \tag{12a}$$

$$\frac{\partial \lambda}{\partial y}=0 \qquad \text{at } y=0, x>0, \text{ for } t>0 \tag{12b}$$

$$\frac{\partial \lambda}{\partial y}=0 \qquad \text{at } y=1, x>0, \text{ for } t>0 \tag{12c}$$

$$\lambda=0 \qquad \text{at } x=x_f, 0<y<1, \text{ for } t>0 \tag{12d}$$

$$\lambda=0 \qquad \text{for } t=t_f, \text{ in } 0<y<1, t>0 \tag{12e}$$

Finally, in this limiting process the following integral term is left:

$$\Delta J[q(x,t)] = -\int_{t=0}^{t_f}\int_{x=0}^{x_f} \lambda(x,l,t)\Delta q(x,t)dxdt \rightarrow 0 \tag{13a}$$

From the hypothesis that $q(x, t) \in L_2(0, x_f)$ $(0, t_f)$, we can write:

$$\Delta J[q(x,t)] = \int_{t=0}^{t_f}\int_{x=0}^{x_f} J'[q(x,t)]\Delta q(x,t)dxdt \tag{13b}$$

Therefore, by comparing equations (13a) and (13b), we obtain the gradient equation for the functional as

$$J'[q(x,t)] = -\lambda(x,l,t) \tag{14}$$

After developing expressions for the search step size $\beta^k$, equation (9), and for the gradient direction $J'[q(x, t)]$, equation (14), we can implement the iterative algorithm of the conjugate gradient method, given by equations (6), until a stopping criterion based on the discrepancy principle described below is satisfied.

**Stopping criterion**

We stop the iterative procedure of the conjugate gradient method when the functional given by equation (5) becomes sufficiently small, that is,

$$J\left[q^{k+1}(x,t)\right] < \varepsilon \tag{15}$$

If the measurements are assumed to be free of experimental errors, we can specify $\varepsilon$ as a relative small number. However, actual experimental data contain measurement errors, which will introduce oscillations in the inverse problem solution as the estimated temperatures approach those measured. Such a difficulty can be alleviated by utilizing the discrepancy principle (Alifanov, 1974) to stop the iterative process, where the number of iterations works as a regularization parameter. In such a principle, we assume that the inverse problem solution is sufficiently accurate when the difference between estimated and measured temperatures is less than the standard deviation ($\sigma$) of the measurements. Thus, the value of the tolerance $\varepsilon$ is obtained from equation (5) as

$$\varepsilon = S\sigma^2 t_f \tag{16}$$

**Computational algorithm**

The basic steps to obtain the solution of the present inverse problem via the conjugate gradient method with adjoint equation are summarized below.

We suppose there is available an estimate $q^k(x, t)$ for the unknown heat flux $q(x, t)$ at iteration k. Thus:

*Step 1*: solve the direct problem given by equations (1) to obtain the estimated temperatures T(x, y, t);

*Step 2*: check the stopping criterion given by equation (15). Continue if not satisfied;

*Step 3*: solve the adjoint problem given by equations (12) to obtain the Lagrange multiplier λ(x, y, t);

*Step 4*: compute the gradient of the functional $J'[q^k(x, t)]$ from equation (14);

*Step 5*: compute the conjugation coefficient $\gamma^k$ from equation (6c) and then the direction of descent $d^k(x, t)$ from equation (6b);

*Step 6*: solve the sensitivity problem given by equations (7) to obtain ΔT(x, y, t), by setting $\Delta q(x, t) = d^k(x, t)$;

*Step 7*: compute the search step size $\beta^k$ from equation (9);

*Step 9*: compute the new estimate $q^{k+1}(x, t)$ from equation (6a) and go to step 1.

**Results and discussion**

We use transient simulated measurements in order to assess the accuracy of the present approach of estimating the unknown wall heat flux q(x, t). The simulated temperature measurements are obtained from the solution of the direct problem for a specified function q(x, t). The temperatures computed in this manner are considered to be errorless, and the simulated measured data is given by:

$$Y = Y_{ex} + \alpha\sigma \tag{17}$$

where $Y_{ex}$ is the solution of the direct problem; α is a random variable with normal distribution, zero mean and unitary standard deviation; and σ is the standard deviation of the measurements. The random variable α is determined with the subroutine DRNNOR from the IMSL (1987).

The direct, sensitivity and adjoint problems were solved with finite-differences by using an upwind discretization for the convection term and an implicit discretization in time. The resultant linear system of equations was solved iteratively by using Gauss-Seidel's method with SOR and red-black reordering (Ortega,1988), so that the computations would be done in vector form in a Cray Y-MP. Such reordering resulted in a speed up of approximately ten over a scalar version of the same computational code.

For the cases considered below, we have taken the total experiment duration ($t_f$) as 0.08 and the channel test-length ($x_f$) as 0.004, while the heat flux at the boundary y = 1 was assumed in the form:

$$q(x,t) = q_x(x) + q_t(t) \tag{18}$$

The domain was discretized by using 101 and 81 points in the x and y directions respectively, while using 41 time steps. Such a number of points was chosen by comparing the solution of the direct problem for the local Nusselt number obtained by finite-differences, with a known analytical solution (Cotta and Ozisik,1986).

By examining equations (12d,e), we note that the gradient of the functional given by equation (14) is null at the final time ($t_f$) and at the final axial position ($x_f$). Therefore, the initial guess used for the iterative process remains unchanged at $t_f$ and at $x_f$. In the examples shown below, we use as an initial guess for the final time and for the final position the exact values for q(x, t), which are assumed available. For other times and axial positions, we take q(x, t) null as the initial guess for the conjugate gradient method. We lose no generality with such approach, since we can always choose $t_f$ and $x_f$ sufficiently greater than the respective experimental time and test section length of interest, so that the boundary heat flux is known.

Figures 2a-2c present the results obtained for a boundary heat flux containing a triangular variation in x and a step variation in time, in the form:

$$q_x(x) = \begin{cases} 1 \text{ , for } x \le 0.001 \text{ and } x \ge 0.003 \\ 1000x \text{ , for } 0.001 < x \le 0.002 \\ -1000x + 4 \text{ , for } 0.002 < x < 0.003 \end{cases} \quad (19a\text{-}c)$$

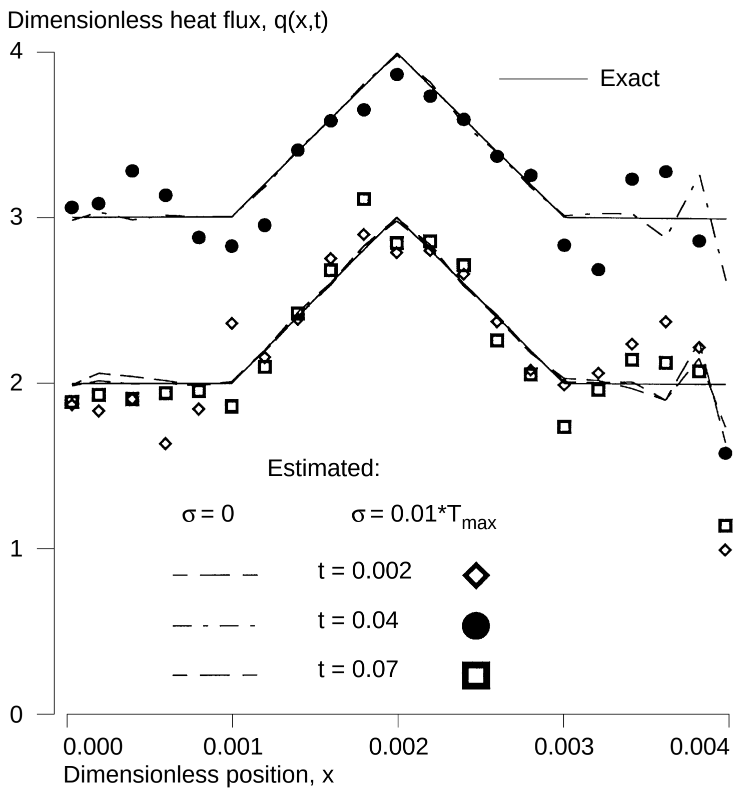


**Figure 2a.**
Inverse problem solution for different times obtained with 21 sensors. Triangular variation in the axial direction given by equations (19)

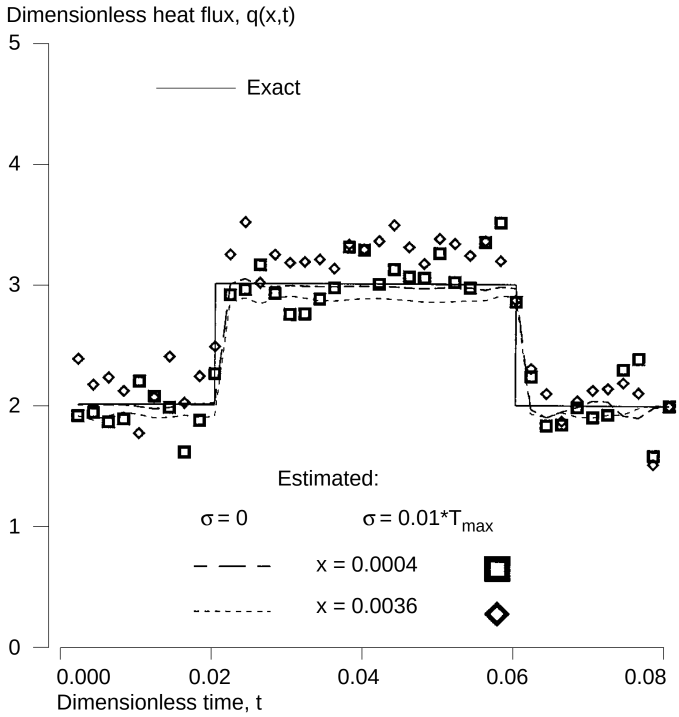


**Figure 2b.** Inverse problem solution for different axial positions obtained with 21 sensors. Step variation in time given by equations (20)

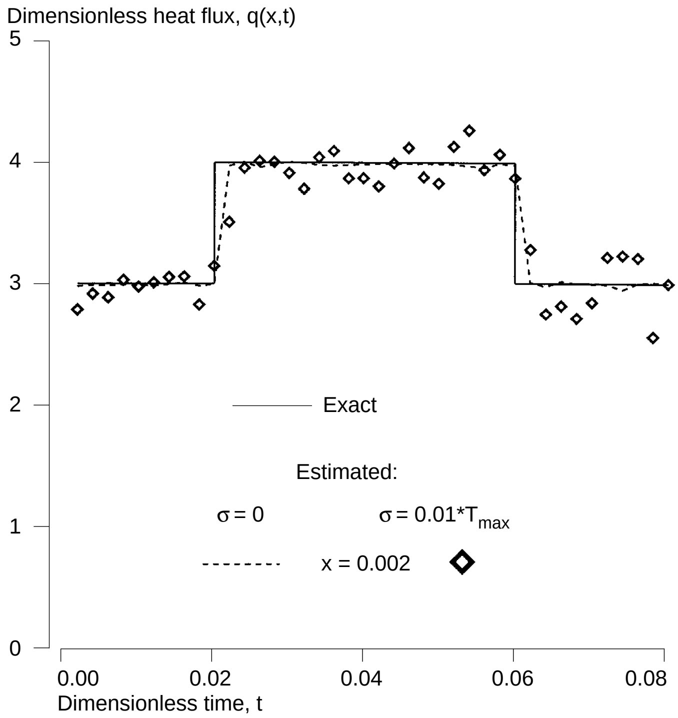


**Figure 2c.** Inverse problem solution for $x$ = 0.002 obtained with 21 sensors. Step variation in time given by equations (20)

$$q_t(t) = \begin{cases} 1\ ,\ \text{for } t \le 0.02 \text{ and } t \ge 0.06 \\ 2\ ,\ \text{for } 0.02 < t < 0.06 \end{cases} \quad (20a,b)$$

For such a case, we have used in the inverse analysis 21 sensors located at y = 0.95. The first sensor is located at $x_1$ = 0.00004 and the last one at $x_{21}$ = 0.00396. The others are equally spaced so that $x_i$ = (i–1)0.0002, for i = 2, …,20. Figure 2 show the results for errorless measurements (dashed lines), as well as for measurements with a standard deviation $\sigma$ = 0.01 $T_{max}$ (symbols), where $T_{max}$ is the maximum temperature measured by the sensors. In Figure 2a, we have the results for the axial variation for three different times, where $q_t(0.002)$ = $q_t(0.07)$ = 1 and $q_t(0.04)$ = 2 from equations (20a-20b). The unknown heat fluxes for such times are accurately predicted, so that the results for t = 0.002 and t = 0.07 fall in the curve at the bottom, while those for t = 0.04 fall in the curve at the top of Figure 2a. The predicted heat flux is in good agreement with the exact one for both errorless measurements and measurements with random error. Figures 2b-2c show the results obtained for the flux variation in time for different axial positions. The results for x = 0.0004 and x = 0.0036 fall on the same curve in Figure 2b as expected, since $q_x(0.0004)$ = $q_x(0.0036)$ = 1 from equations (19). The results shown in Figure 2c for x = 0.002, where q(x, t) has a peak in x, are also in good agreement with the exact functional form assumed for q(x, t).

The RMS error ($e_{RMS}$) for the results shown in Figures 2 obtained with errorless measurements, is 0.014. We define the RMS error here as:

$$e_{RMS} = \frac{1}{N}\sqrt{\sum_{i=1}^{N}[q_{ex}(x_i,t_i) - q_{est}(x_i,t_i)]^2} \quad (21)$$

where N is the total number of measurements used in the inverse analysis, while $q_{ex}$ and $q_{est}$ are the exact and estimated heat fluxes respectively.

Figures 3a-3c present the results obtained for a heat flux with a step variation in x and with a triangular variation in time, in the form:

$$q_x(x) = \begin{cases} 1\ ,\ \text{for } x \le 0.001 \text{ and } x \ge 0.003 \\ 2\ ,\ \text{for } 0.001 < x < 0.003 \end{cases} \quad (22a,b)$$

$$q_t(t) = \begin{cases} 1\ ,\ \text{for } t \le 0.02 \text{ and } t \ge 0.06 \\ 50t\ ,\ \text{for } 0.02 < t \le 0.04 \\ -50t + 4\ ,\ \text{for } 0.04 < t < 0.06 \end{cases} \quad (23a\text{-}c)$$

where the dashed lines show the results obtained with errorless measurements and the symbols show the results obtained with measurements with a standard deviation of $\sigma = 0.01T_{max}$. The 21 sensors used for this case are located at y = 0.95 and at the same axial positions as for the case shown in Figures 2. Figure

3a shows the axial variation of q(x, t) for different times that correspond to $q_t(t)$ = 1, as given by equations (23). Similarly, Figure 3b shows the axial variation of q(x, t) for t = 0.04, when $q_t(t)$ has a peak, i.e. $q_t(t) = 2$ as given by equations (23). In Figure 3c, we have the results for the variation of q(x, t) in time for three different axial positions, so that, in accordance with equations (22), we have $q_x(0.0004) = q_x(0.0036) = 1$ and $q_x(0.002) = 2$. As for the case presented in Figures 2, Figures 3 show that present function estimation approach is capable of recovering the unknown heat flux q(x, t) quite accurately for errorless measurements, as well as for measurements containing random errors. The RMS error is 0.045 for the results shown in Figures 3, obtained with errorless measurements.

The results shown above in Figures 2 and 3 can be generally improved by using more measurements in the inverse analysis. Let us consider, for example, the estimation of the axial variation of q(x, t) shown in Figure 3a. In Figure 4, we present the estimation of q(x, t) for the same case studied in Figure 3a, but using the errorless measurements of 101 sensors instead of 21. The sensors are equally spaced along the channel length and at y = 0.95. The time frequency of measurements was considered to be the same as for Figure 3a. By comparing Figures 3a and 4, we can clearly notice the improvement in the estimation of

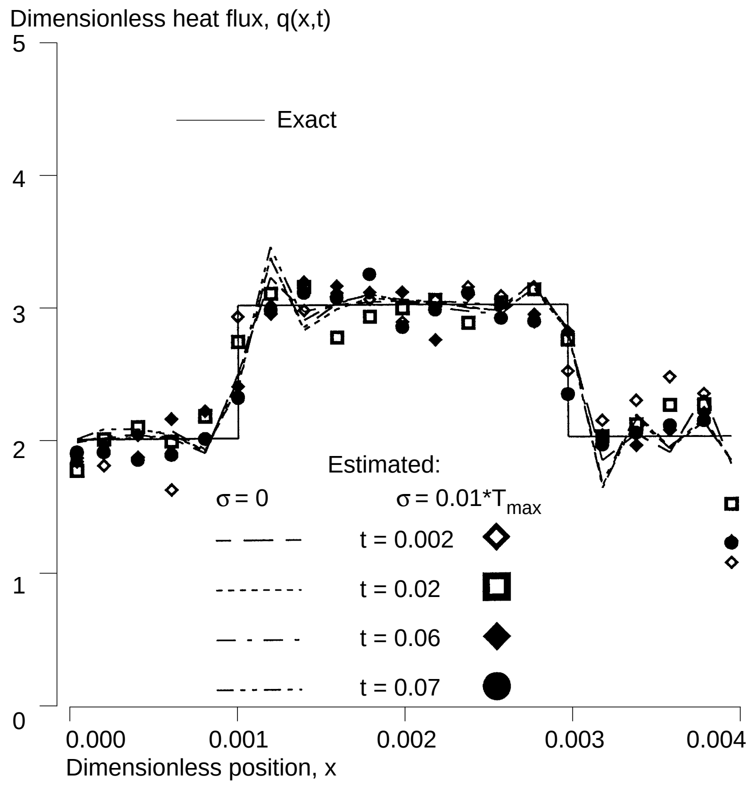


**Figure 3a.** Inverse problem solution for different times obtained with 21 sensors. Step variation in the axial direction given by equations (22)

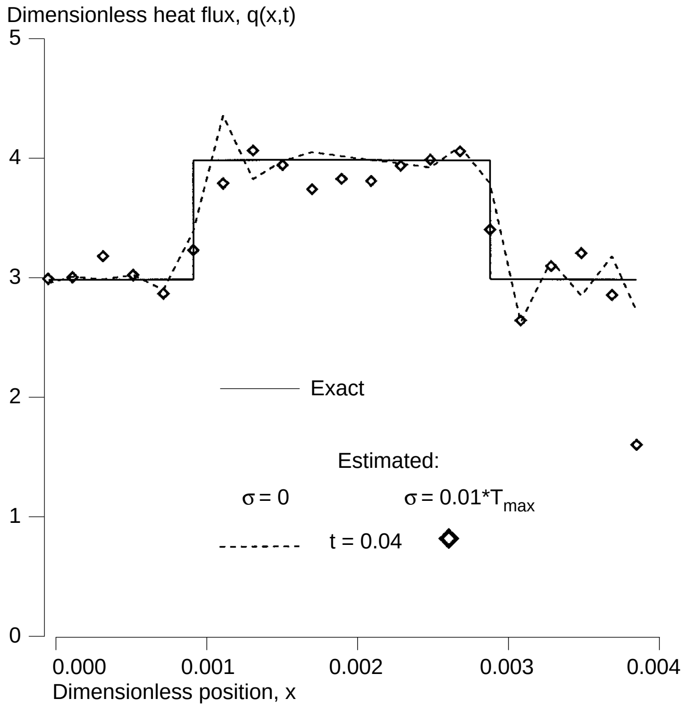


**Figure 3b.**
Inverse problem solution for *t* = 0.04 obtained with 21 sensors. Step variation in the axial direction given by equations (22)

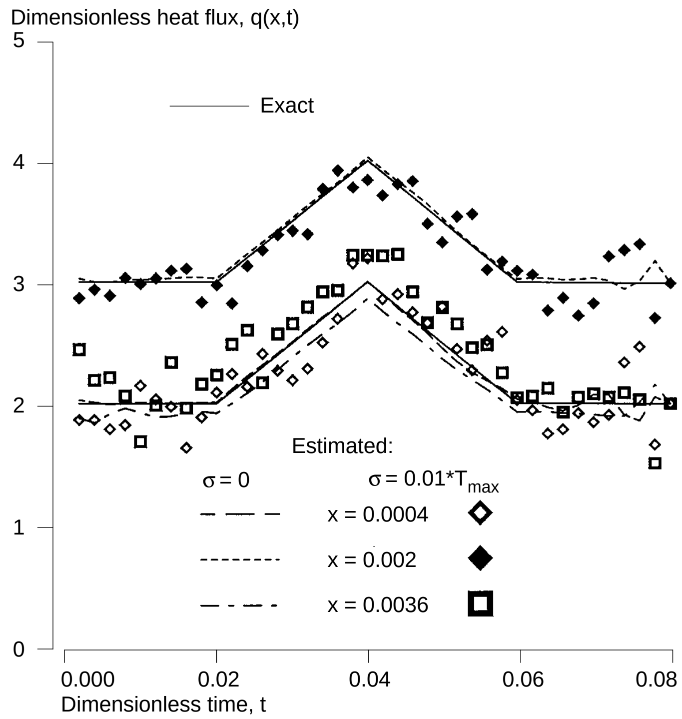


**Figure 3c.**
Inverse problem solution for different axial positions obtained with 21 sensors. Triangular variation given by equations (23)

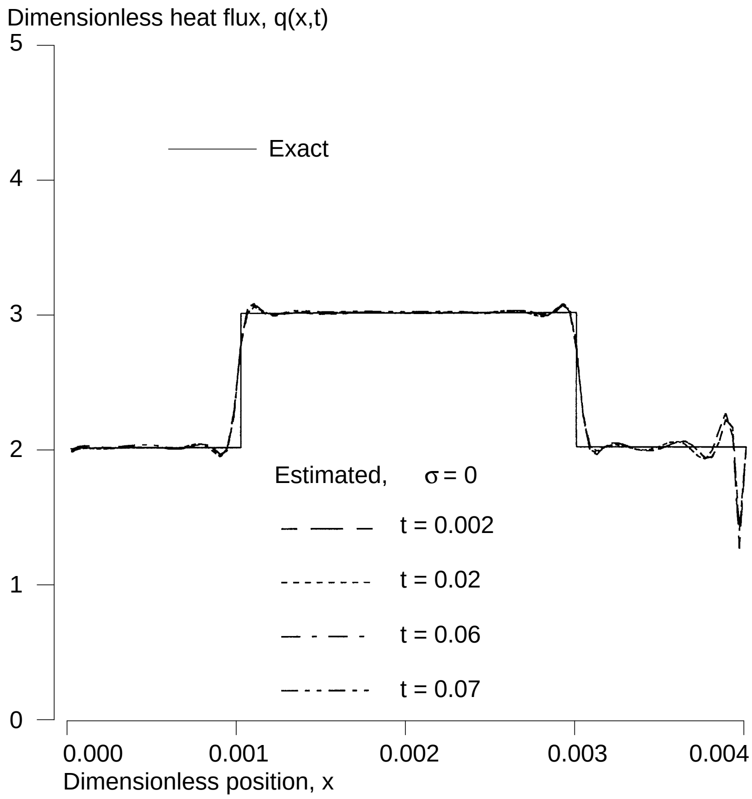


**Figure 4.**
Inverse problem solution for different times obtained with 101 sensors. Step variation in the axial direction given by equation (22)

q(x, t) by using more sensors along the channel. The RMS error obtained with 101 sensors is 0.013 as compared to 0.045 obtained by using 21 sensors.

For inverse heat conduction problems dealing with the estimation of a boundary condition, the sensors should be located as close to the boundary with the unknown condition as possible (Beck *et al.*, 1985) in order to improve the estimation. Such is also the case for inverse convection problems. We have estimated q(x, t) for $q_x(x)$ and $q_t(t)$ given by equations (22) and (23), respectively, and by using the errorless measurements of 21 sensors located at the same axial positions as for Figures 3, but at y = 0.9, instead of at y = 0.95. The RMS error has increased to 0.238, as compared to 0.045 obtained with the sensors located at y = 0.95.

We note in Figures 2-4 that generally the agreement between the estimated solutions and the exact functional form assumed for q(x, t) tends to deteriorate near the final axial position and near the final time. This is due to the very small values of the gradient of the functional, equation (14), in such regions as can be noticed by examining equations (12d,e).

## Conclusions

A function estimation approach based on the conjugate gradient method with adjoint equation has been successfully applied to the inverse problem of estimating the timewise and spacewise variation of the wall heat flux in a parallel plate channel.

Results obtained with simulated measurements show that the present approach is capable of recovering sharp corners and discontinuities in the exact functional form assumed for the unknown heat flux. The results appear to be stable with respect to random measurement errors.

We note that the sensors should be located as close to the boundary with the unknown heat flux as possible, in order to obtain accurate estimations. Also, as many sensors as possible should be used for the inverse analysis, without disturbing the flow in the channel.

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